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Towards Probabilistic Quantitative Precipitation WSR-88D Algorithms: Preliminary Studies and Problem Formulation: Phase 3

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EXECUTIVE SUMMARY

The report describes progress made towards developing a scientifically rigorous methodology for operational probabilistic quantitative precipitation estimation (PQPE) for hydrologic applications. The methodology will be based on the WSR-88D measurements complemented with rain gauge and satellite data. It is flexible enough to allow a smooth transition to the polarimetric era after the planned upgrade of the operational network of radars. The overall strategy is to demonstrate hydrologic utility of the probabilistic information of the precipitation estimates. This involves two major elements (1) developing a theoretical and operational framework for probabilistic radar-rainfall estimation; and (2) connecting the PQPE input with a hydrologic application. This report documents initial progress made in both elements.

The authors define a radar PQPE product as a set of situation-dependent parameter values in a model describing the probability distributions of the uncertainties in the radar-estimated rainfall. The distributions quantify the available probabilistic knowledge about the true spatial rainfall that is likely, given current radar measurements and other available information. The model parameter values determine unambiguously the uncertainty distributions for each operationally useful distance from the radar and spatiotemporal averaging scale. This allows generating different user-specific outputs demanded by various operational applications. Among these outputs are the uncertainty bounds and probabilities of exceedence. Generating an ensemble of the probable rainfall maps to provide the input for the ensemble forecasting schemes is also possible. The report presents early results of the model formulation.

The hydrologic utility of the PQPE methodology will be demonstrated using the flash flood forecasting problem. This part of the project is performed in close collaboration with the Hydrologic Research Center (HRC). The demonstration is limited geographically to the Oklahoma region. This report documents developments leading to PQPE application in a Flash Flood Guidance and Monitoring system. The authors present uncertainty analysis of the Thresh-R model which is the basis for FFGM and of the soil moisture accounting model. The early results also include effect of the uncertainty in the rainfall input via an example of ensemble PQPE simulation.

Useful Acronyms

ABRFC	Arkansas Basin River Forecast Center
CDP	conditional distribution of precipitation
CSI	Critical Success Index
DHR	Digital Hybrid Scan Reflectivity
FAR	False Alarm Ratio
FFG	Flash Flood Guidance
FFMP	Flash Flood Monitoring and Prediction
HL	Hydrologic Laboratory
HRC	Hydrologic Research Center
KDP	specific differential phase shift
KINX	WSR-88D in Tulsa, OK
KTLX	WSR-88D in Twin Lakes, OK
MAP	Mean Area Precipitation
MFB	Mean-field-bias
MPE	Multisensor Precipitation Estimation
NSSL	National Severe Storms Laboratory
NWS	National Weather Service
PED	product-error-driven
POD	Probability of Detection
PPS	Precipitation Processing System
PQPE	Probabilistic Quantitative Precipitation Estimation
RFC	River Forecast Center
SOW	Statement of Work
WFO	Weather Forecast Office
WSR-88D	Weather Surveillance Radar - Doppler
Z	radar reflectivity
ZDR	differential reflectivity
CSSA	Convective/Stratiform Separation Algorithm
REC	Radar Echo Classifier
RCA	Range Correction Algorithm
EPPS	Enhanced PPS
HCA	Hydrometeor Classification Algorithm
EPRE	Enhanced Preprocessing

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A. BACKGROUND INFORMATION

Hydrologic forecasting and water resources services performed for the public by the National Weather Service (NWS) require decision making in presence of uncertainty due to limitation in our understanding of nature, available information, and predictability of natural phenomena. High space and time resolution precipitation estimates are the main input for many of the forecasting models. These estimates are based on information from the network of weather radars WSR-88D combined with rain gauge and satellite data (e.g. Fread et al. 1995; Stallings and Wenzel 1995). The current operational NWS multi-sensor rainfall algorithms produce only deterministic fields of precipitation intensity and accumulations (e.g. Fulton et al. 1998). However, users would be better able to make informed decisions if they knew not only the best rainfall estimate but also the associated uncertainty and/or range that most likely includes the actual amount of that occurred.

The Office of Hydrologic Development of the NWS intends to address this shortcoming of the existing algorithms by preparing a comprehensive plan for development of a new generation of algorithms for the precipitation estimation. These algorithms are referred to as *probabilistic quantitative precipitation estimation*, or PQPE. Krajewski and Ciach (2003) developed a comprehensive plan for nation-wide development of the PQPE algorithms. Their report lays out an early formulation of the problem, identifies conceptual, methodological and technological issues, and proposes a feasible plan of action. However, because the plan calls for considerable expenditures of resources, the PQPE Advisory Team suggested preceding it with a geographically focused effort of an end-to-end demonstration of the utility of the PQPE approach. In response, Krajewski et al. (2003) formulated a plan for developing such a demonstration.

In this report we address early results towards developing a demonstration of the utility of the PQPE for flash-flood forecasting. We describe (1) a formulation of the radar-only PQPE algorithm and plans for identifying appropriate model and estimating its parameters based on actual data from Oklahoma; (2) software setup for the above; and (3) simulation based studies of the flash-flood guidance designed towards a comprehensive treatment of uncertainty including the PQPE.

B. FORMULATION OF THE PQPE METHODOLOGY

During the Phase II of this project, we continued our analysis and refinement of the methodological framework for the PQPE problem that was initiated in Phase I (Krajewski and Ciach 2003). For the completeness of this report, we briefly summarize the proposed methodology for the PQPE algorithm development as we understand it now.

B.1. Basic Definitions

The four fundamental notions defined below are used throughout this report:

- *True rainfall*: The amount of rain-water that has fallen on a specified area in a specified time-interval.
- *Radar-rainfall (RR)*: An approximation of the true rainfall based on radar data corresponding to the same spatio-temporal domain.
- *RR uncertainties*: All systematic and random differences between RR and the corresponding true rainfall.
- *Ground reference (GR)*: Estimates of the area-averaged rainfall accumulations based on rain-gauge data that are used to evaluate RR products.

B.2. Problem Description

The progressive evolution of the operational RR products has been guided by the attempts to quantify and to reduce the uncertainties in the RR estimates. The currently existing RR maps produced operationally by the NWS (the Stage II and III products) are just arrays of numbers describing the spatial distribution of approximate rainfall accumulation values that are obtained based on the WSR-88D reflectivity measurements corrected with the available concurrent rain-gauge data. Application of the term "quantitative precipitation estimates" QPE to such products implies that the maps are completed with quantitative information about the product uncertainties. Without such information about the relation of the RR product to the corresponding true rainfall, both the notion of "quantitative" and the mathematical term "estimation" would be meaningless in this context. However, despite a wide use of this term, the operational QPE products are devoid of their uncertainty information. We believe that the development of the probabilistic quantitative precipitation estimation (PQPE) products based on sound empirical evidence will be the optimal comprehensive solution for this pathological situation.

The probabilistic products, both in meteorology and hydrology, convey the inferred information about the unknown true value of a physical quantity in terms of its probability distribution rather than its one "best" estimate (e.g. Krzysztofowicz 2001). Thus, the radar PQPE product can be mathematically defined through the conditional probability distributions of the likely true rainfall, given the current radar measurements and other available information. These distributions can be determined by specific parameter values of a general uncertainty distribution model developed in this project. The model parameters have to determine unambiguously the uncertainty distributions of given RR estimates in different rainfall regimes for each operationally useful distance from the radar and spatio-temporal averaging scale. From such a general PQPE product, one can directly derive any specific uncertainty characteristics (for example, the RR expectation, standard errors, probabilities of exceedence, or an ensemble of probable rainfall maps) that can be required for different operational applications.

B.3. Basic Requirements

During the discussions with the panel of experts engaged in the Phase I of this project (Krajewski and Ciach 2003), it was agreed that any method that will be applied to generate the PQPE products has to satisfy several key requirements. These requirements were further analyzed and refined in the course of the Phase II of the project. We summarize them briefly below:

1. The method has to be empirically "verifiable." Conditions have to be assured to systematically evaluate the degree of agreement between the PQPE results and the RR uncertainties estimated based on reliable GR in selected "validation sites."

2. The method has to be adjustable to different synoptic and topographical situations, and to the changing operational environment, by its model parameter calibration using available information.

3. The method has to account for the spatio-temporal dependencies in the errors process to provide the PQPE products over a broad range of spatial and temporal scales used in different hydrological applications.

4. The method has to work with the current reflectivity-only WSR-88D algorithms, the multi-parameter (MPE) algorithms using the available concurrent rain-gauge and satellite data, and the polarimetric algorithms (using differential reflectivity and differential phase-shift) available operationally after the upcoming upgrades of the WSR-88D radars.

5. The method has to provide the PQPE products in a format appropriate for their efficient usage in different hydrological applications.

B.4. Development of the PQPE Algorithm

During the previous phases of this project, it has been agreed that the product-error-driven (PED) modeling approach for the PQPE algorithm, described in our reports for the Phase1 and Phase 2, will be developed using a fully empirically-based framework. This decision acknowledges the obvious fact that building a PQPE algorithm has to be preceded by the development of a realistic and parsimonious mathematical model of RR uncertainties underlying the probabilistic nature of RR products. Only a thorough and comprehensive data analysis can result in the identification of such a realistic model suitable for the PQPE applications.

However, the actual exploratory data analysis of the PPS products has not started yet because the preparation of the data sample has not been completed. Therefore, the algorithm development efforts in the Phase 3 concentrated on the following tasks:

1. Development of the PED modeling methodology.
2. Preparation of the large data sample.
3. Testing the GR error filtering method.

Below, we briefly describe the work performed in these tasks and the results that have been achieved so far.

B.4.1. Development of the PED Modeling Methodology

There are many sources of errors in RR products and we discussed them in the Phase 1 report. The PED approach focuses on the combined effect of all the errors, its modeling, and estimation of the model parameters. This follows the fact that one cannot delineate the separate effects using the available measurable quantities. In practice, only the combined effect on the RR estimates can be measured and quantified. Our objective is to create a flexible parameterized mathematical model of the relation between the RR product values and the corresponding True Rainfall conditioned on different situations. The four conditions that we plan to quantify are the distance from the radar, space-time averaging scale, rainfall regime, and the PPS setup. In the PQPE algorithm, this model will be used to quantify the probability distributions of the probable True Rainfall, given the RR value and the other abovementioned conditions.

B.4.1.1. *General Structure of the Model*

The relationships between RR and the corresponding truth can be described by the family of the conditional bivariate frequency distributions that we call the “true verification distributions” (TVD):

$$(R_a\,,\,R_r)_{L,T,d,S} \;=\; f(R_a\,,\,R_r \mid L,\,T,\,d,\,S) \qquad (1)$$

where R_a and R_r are the corresponding (concurrent and collocated) True Rainfall and RR values, respectively, L is the spatial averaging scale, T is the temporal scale (accumulation interval), d is the distance from the radar station, and S denotes the type of the precipitation system (rain regime). In principle, these distributions can be retrieved from the radar-gauge data samples, if additional information on the rainfall variability is available (see section B.4.3 below).

To simplify the notation, we can focus on one spatiotemporal resolution (L, T), distance (d) from the radar and rain regime (S). To model the (R_a , R_r) distribution for these specified conditions, it is convenient to use the following functional-statistical representation:

$$R_a = h(R_r)\, e(R_r) \qquad (2)$$

where $h(\cdot)$ is a deterministic distortion function and e is a random variable representing the random uncertainties. If parametric models of the $h(R_r)$ and $e(R_r)$ functions are identified and its parameter estimates conditioned on a specific situation are known, this representation prescribes the way in which the ensembles of probable True Rainfall values, or only its selected statistical characteristics required by the users, can be generated for each given value of RR.

B.4.1.2. *Main Elements of the Model*

Because all systematic biases can be described by the deterministic distortion function, we can assume without any loss of generality that the mean of the random uncertainty factor is always equal to unity ($E\{e\}=1$). This allows the rigorous definition of the $h()$ function based on the general regression formula:

$$h(x) = E\{R_a \mid R_r=x\}, \tag{3}$$

which, in practice, can be identified and estimated using any version of the nonparametric regression apparatus (Hardle 1990; Simonof 1996).

Although the mean of the multiplicative random uncertainty factor, $E\{e(R_r)\}$, is equal to unity for each value of R_r, its distribution can vary with R_r. The first step in identifying this dependence is to estimate the $e(R_r)$ variance as a function of R_r. This can be done in the similar way as estimating the $h(R_r)$ function. An example of such a procedure is shown in the section B.4.1.3 below. Next, we have to find a suitable parametric model for the $e(R_r)$ distributions. This can be achieved based on extensive data analysis by examining the shapes of the actual $e(R_r)$ distributions under different values of the conditioning factors. Since the extreme rainfall events are the most important in hydrological practice, it is essential that the selected probability distribution model describes the uncertainty distribution tails with a reasonable accuracy. Examples of the models that have distinctly different tails are the gamma, lognormal and beta distributions. Each of them can lead to different decisions based on the PQPE results. The goodness-of-fit of these and several other models will have to be tested on the large data sample before a justified choice can be made.

Once parametric models of the $h(R_r)$ function and the $e(R_r)$ variable are identified, the dependence of their parameters on the averaging scale (L, T), distance from the radar (d) and rain regime (S) can be estimated based on the family of the verification distributions (estimated bivariate distributions of RR and the corresponding True Rainfall):

$$(R_r\,,\, R_a)_{Ln,Tn,d,S}\,,\;\; n=1,\, 2,\, \ldots\,,\, N_{max} \tag{4}$$

where the distributions are sampled for several spatio-temporal scales that are multiples of the original RR product scale. Spatio-temporal dependencies in the model parameters can then be modeled to reproduce the dependence of these conditional model parameter estimates on the discrete series of scales (A_n ,T_n), for each given distance from the radar (d) and precipitation regime (S).

It is still unclear to us how to stratify the data sample according to the precipitation regime (S) so that this information is meaningful for the PQPE methodology. The appropriate classification has to be based on data that are readily operationally available during the PPS processing, preferably the radar data. In addition, it should exhibit distinct differences in the PED model parameter values for the different regimes. One of such classification schemes by Steiner et al.

(1995) has been investigated by Ciach et al. (1997). Our results indicated that its effects on the RR estimation algorithm are practically the same as the stratification of the data sample according to different RR values. Consequently, using this specific precipitation regime classification can only complicate the PQPE algorithm unnecessarily without adding any value to it. A classification of the synoptic situation, or some other information (e.g., the zero isotherm level) based on the operational weather forecasts could perhaps be a better alternative to the schemes using the radar data only. However, the best way to use this external information remains to be investigated. We hope that the PQPE Advisory Team will help us with this issue.

Obviously, the successful development of operationally applicable parametric models of the $h(R_r)$ function and the $e(R_r)$ variable will most likely require a number of generalizations and simplifications in the mathematical description of the abovementioned dependences. The specific formulas will have to be identified during the planned extensive analysis of large data samples.

B.4.1.3. *Preliminary Analysis of RR Error Structure*

In the Phase 3 of this project, we performed a preliminary study of the basic elements of the RR error structure. It is an extension of our first analysis that we described in the Phase 1 report. It is based on a relatively small data sample of 50 rainy days. The radar data from the Tulsa, Oklahoma, NEXRAD station (KINX) were quality controlled and converted into hourly accumulations in polar grids over 23 surrounding rain gauge stations (Vignal and Krajewski 2001). Using this data sample, we estimated the deterministic distortion function, $h(\cdot)$, and the variance of the multiplicative error factor, $e(\cdot)$, as functions of the accumulation time. At this stage, we did not consider the dependences on the spatial scale, distance from the radar, or the rain regime. The temporal dependences in the RR error process were estimated for five accumulation intervals: 1, 3, 6, 12, and 24 hours.

The deterministic distortion function was estimated using the following scheme of moving-window averaging:

$$h(r) \approx \langle R_r \mid r-u \leq R_g \leq r+u \rangle \qquad (5)$$

where R_g is the rain gauge rainfall accumulation and u is the averaging window size. The window size was increased with R_g to compensate for the decreasing number of data points. These functions, for the five time scales, are shown in Figure 1.

The results in Figure 1 show that the systematic distortion component is a nonlinear function of the True Rainfall. This function is a way to quantify the conditional biases in different RR products that have been qualitatively demonstrated long time ago by Austin (1987) and investigated using an idealized analytical model by Ciach et al. (2000). For the larger accumulation intervals (6, 12 and 24 hours), these conditional biases are relatively small and invariant in respect to the time-scale. The outstanding results for the 1-hour and 3-hour time-scales might be the effect of large rain gauge representativeness errors, however, they might as well indicate a distinctly different uncertainty structure at the short scales. This question requires more extensive analyses using the area-point error filtering method described in section A.1.3 below.

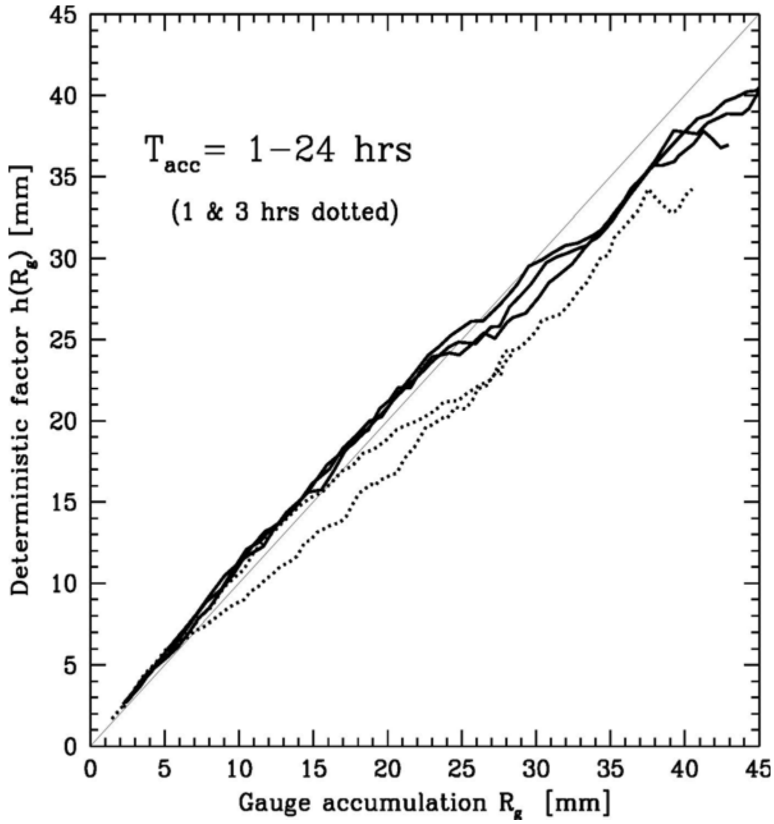

Fig. 1. Systematic distortions, *h*, as functions of True Rainfall for the five accumulation intervals.

The standard deviation of the random error factor as a function of the rain gauge rainfall accumulation for the same five accumulation intervals is shown in Figure 2. The $e(\cdot)$ variances, σ_e^2, as a function of R_g were estimated in a similar way as the $h(\cdot)$ functions:

$$\sigma_e^2(r) \approx < (e(R_g)-1)^2 \mid r-u \leq R_g \leq r+u > \quad (6)$$

using the same moving-averaging scheme with variable window size.

The results in Figure 2 show that, for each of the five accumulation intervals, the standard deviation of the multiplicative random uncertainty factor decreases rapidly with increasing rainfall and then stabilizes at the level of about 30%. The estimates of the random component seem to be less sensitive to the shorter time-scales than the estimates of the systematic distortion function. This invariance, if confirmed on the large data sample that we currently prepare, can be a good basis to reduce the number of parameters of the final PED model that will be used for the PQPE algorithm.

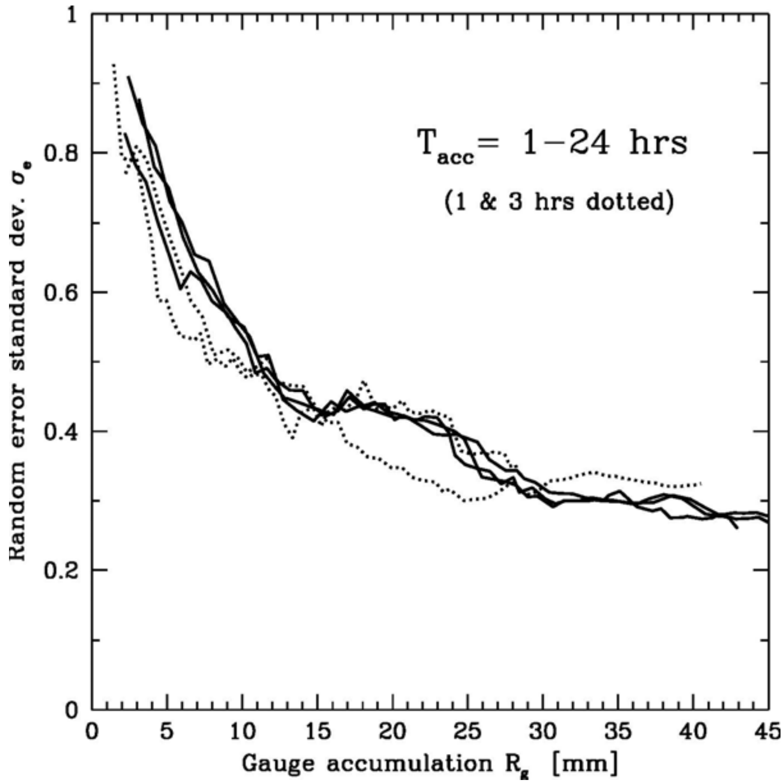

Fig. 2. Standard deviations of the multiplicative random error factor, *e*, as functions of True Rainfall for the five accumulation intervals.

B.4.1.4. *Two Idealized Implementations*

One of the most difficult elements of the PQPE methodology concerns including the spatial and temporal dependences that exist in the RR uncertainty process into the probabilistic model that we create using the PED approach. In the course of developing a viable technique to treat this problem, we started with two idealized implementations of the PQPE algorithm that are based the following simple model of the (R_a , R_r) distribution:

$$R_a = R_r \, e \tag{7}$$

where the random uncertainty factor, *e*, is lognormally distributed and does not depend on the RR value (a multiplicative homoscedastic model). Both algorithms are designed to simulate ensembles of the probable true rainfall conditioned on the RR values obtained from the radar data. The first algorithm includes the temporal dependences in the uncertainty process, whereas the second algorithm generates ensembles of spatially correlated uncertainty fields. These dependences in the *e* variable were modeled based on a meta-Gaussian model that starts with a time series (or spatial field) of the uncorrelated standard normal white noise. Next, this variable is correlated in time (or space) using weighted moving-window averaging with a specified averaging mask, and the outcomes are transformed into the positively defined random process through the exponential transformation. The parameters of this transformation are such that the resulting variable has the mean equal to unity and the specified variance.

The temporally correlated 1-D version of this simple PQPE algorithm was applied to generate the ensembles of time-series of probable true rainfall for the lumped flash flood forecasting model that we implemented and upgraded into probabilistic framework with the help of the Hydrologic Research Center. This numerical experiment is a part of the PQPE project and is described in section A.2 of this report. The flow-chart of this algorithm consists of the following steps:

1. Generate an 1-D array of independent standard normal deviates.
2. Apply the weighted moving-window averaging to the array.
3. Apply the exponential transformation to the smoothed array.

This procedure generates one time-series of the time-correlated meta-Gaussian uncertainty process. It is repeated to obtain the required number of the ensemble members. The weighted moving-window averaging can, in principle, be performed using any averaging mask. This allows fairly flexible adjustment of the simulated temporal uncertainty structure to the empirical estimates that we will obtain from the data analysis. At this stage, we used a simple polynomial mask for this smoothing. The parameters controlling the algorithm outcomes are:

1. The number of realizations in the ensemble.
2. The length of the simulated time-series.
3. The size and shape of the smoothing mask.
4. The standard deviation of the uncertainty factor.

The 1-D algorithm described above is fast and enables generating large ensembles consisting of 10^5, or more, realizations for the probabilistic hydrological forecasting model. However, it can only be applied to the lumped rainfall-runoff models.

The spatially correlated 2-D version of the PQPE algorithm was applied to generate the ensembles of spatial fields of probable true rainfall. It can be applied to a distributed hydrological forecasting model, or used to compute the RR uncertainty bounds for different spatial scales. The flow-chart of this algorithm consists of the following steps:

1. Generate a 2-D array of independent standard normal deviates.
2. Apply the weighted moving-window averaging to the array.
3. Apply the exponential transformation to the smoothed array.

This procedure simulates one field of the space-correlated meta-Gaussian uncertainty process and is repeated to generate the required number of the ensemble members. The weighted moving-window averaging can, in principle, be performed using any averaging mask. It can also be adjusted to the empirical estimates that we plan to obtain from the data analysis. The parameters controlling the algorithm outcomes are:

4. The number of realizations in the ensemble.
5. The size of the simulated spatial array.
6. The size and shape of the smoothing mask.
7. The standard deviation of the uncertainty factor.

The 2-D algorithm described above is computationally demanding and the largest ensembles that we generated so far consisted of up to 10^3 realizations. However, it can be used in a much broader range of applications than the 1-D algorithm. Therefore, it will be the basis for the development of the full PQPE algorithm. During its further development, we will extend the simulation procedure to include the dependences of the uncertainty factor on the RR values and the distance from the radar, and implement different probability distribution models.

B.4.2. GR Error Filtering

We developed a conditional distribution transformation (CDT) method for improving RR uncertainty analyses that use sparse rain gauge networks as the ground reference. The objective of the CDT method is perform a conditional point-area rainfall distribution transformation in order to filter out the rain gauge representativeness errors from radar-rain gauge samples The application of the rain gauge error filtering is essential for the estimation of the spatial dependences in the PQPE model because large differences between the sampling areas of radar and rain gauge measurements can render the results of direct comparisons meaningless. We tested the validity and evaluated the accuracy of the CDT method. The tests were based on the data from the ARS Micronet. A detailed description of the CDT method and its tests has been documented in Habib at al. (2004). Below, we present only an outline of this effort and its results.

B.4.2.1. *Point-Area Distribution Transformation Method*

Our implementation of the point-area transformation scheme follows in principle the methodology presented in Morrissey (1991). Let R_p represent point (single rain gauge) rainfall with mean $E\{R_p\}$ and variance $Var\{R_p\}$, and R_a represent the rainfall averaged over an area A with mean $E\{R_a\}$ and variance $Var\{R_a\}$. The means of the two corresponding processes are equal, i.e. $E\{R_a\}=E\{R_p\}$, and the variances can be related to each other based on the spatial correlation in the rainfall field in the following way:

$$Var\{R_a\} = \frac{Var\{R_p\}}{A^2} \int_A \int_A \rho(x,y) dx^2 dy^2 \tag{8}$$

Now, given the probability distribution of R_p, we want to estimate the distribution of R_a that has the same mean as R_p, but different and known variance. As an approximate solution for this problem, we adopted a nonparametric distribution transformation method proposed by Journel and Huijbregts (1978).

The probability distribution of rain gauge measurements R_p can always be represented using a transformation that expresses R_p as a function of the standard normal random variable $R_p=\phi_{Rp}(u)$, where u is the standard Gaussian variable and the equality is in the sense of the same probability distributions (Journel and Huijbregts 1978). This function is approximated using a decomposition (expansion) based on Hermite polynomials:

$$\phi_{R_p}(u) \approx \sum_{i=0}^{n} \frac{\psi_i}{i!} H_i(u),$$

where $H_i(\cdot)$ are Hermite polynomials of the order i and ψ_i are their expansion coefficients, and the first four Hermite polynomials are shown as an example. The decomposition coefficients are fitted to the empirical frequency distribution of R_p using an iterative procedure described in Journel and Huijbregts (1978). The coefficients ψ_i are related to the mean and variance of the point rainfall as follows:

$$\psi_o = E\{R_p\},$$

$$\sum_{i=1}^{n} \frac{\psi_i^2}{i!} = Var\{R_p\}.$$

The main assumption of the point-area transformation scheme proposed by Journel and Huijbregts (1978) is that the function ϕ_{Ra}, expressing the areal rainfall as a function of the standard normal random variable (just as ϕ_{Rp} represents the point rainfall), has the same Hermite expansion as ϕ_{Rp}, but its decomposition coefficients are modified by a single scaling factor, a, in the following way:

$$\phi_{R_a}(u) \approx \sum_{i=0}^{n} \frac{\psi_i a^i}{i!} H_i(u),$$

where the coefficients ψ_i are the same as in the point rainfall decomposition. Note that this distribution transformation preserves the distribution mean since $a^0=1$. On the other hand, the variance of the transformed distribution of the areal rainfall can now be expressed as:

$$Var\{R_a\} = \sum_{i=1}^{n} \frac{\psi_i^2 a^{2i}}{i!},$$

and thus, it depends on the known decomposition coefficients ψ_i of the rain gauge rainfall and the scaling factor, a, only. This equation is a monotone function of a. Thus, if the variance of R_a is known, the scaling factor can be determined, using any iterative or graphical method, so that this equality is fulfilled.

Given the estimates of the coefficients, ψ_i, and the scaling factor, a, the computer generated standard normal deviates can be substituted into the Hermite expansion to simulate the distribution of the areal rainfall. This point-area transformation procedure is general. It can be applied to the whole data sample, as well as to its sub-samples selected (conditioned) in any specific way. Since our focus in this study is on quantification of RR uncertainties, the distributions and their transformation have to be conditioned on the radar estimates, R_r.

The scheme of this conditional distribution transformation (CDT) can be summarized as follows. First, the rain gauge rainfall values in the data sample are grouped into sub-samples that are conditioned on a number of ranges of the RR values, $(R_p|R_r=r)$, each range centered on a RR value, r. The number of the sub-samples and their sizes depend on the amount of available data. Then, the correlation function of the point rainfall conditioned on the radar estimate value, $(\rho|R_r=r)$, is estimated. This enables the estimation of the conditional variances of areal rainfall, $Var\{R_a|R_r=r\}$. For each of the sub-samples $(R_p|R_r=r)$, the conditional coefficients, $(\psi_i|R_r=r)$, of the Hermite polynomial decomposition and the conditional scaling factors, $(a|R_r=r)$, are estimated. Finally, the conditional distribution transformation functions $(\phi_{Ra}|R_r=r)$ are computed and used to generate values that correspond to the areal rainfall $(R_a|R_r=r)$. These generated values can then be used to provide the desired estimates of the conditional distributions of the true area-averaged rainfall, $f(R_a|R_r)$, conditioned on RR. They can be applied to reconstruct the bivariate distribution of RR and the corresponding true areal rainfall based on the following formula:

$$f(R_a , R_r) = f(R_a|R_r)\, f(R_r) ,$$

which can then be used to identify the PQPE model at different spatiotemporal scales as outlined in the previous Section of this report.

B.4.2.2. *Tests of the CDT Method*

The goal of the point-area transformation scheme is to obtain the estimates of conditional probability distributions of the true areal rainfall, conditioned on RR values, based on the conditional distributions of rain gauge rainfall and information on the conditional spatial correlation in the rain-field.

To evaluate the performance of the CDT method we used the data sample of point rainfall, areal rainfall and the corresponding RR estimates over three testing boxes within the ARS Micronet that are indicated in Figure 3.

Only one time scale of 15 minutes was considered at this stage. We stratified the sample into sub-samples of four intervals of the 15-minute RR values, R_r. For each sub-sample separately, we carried out the following procedure:

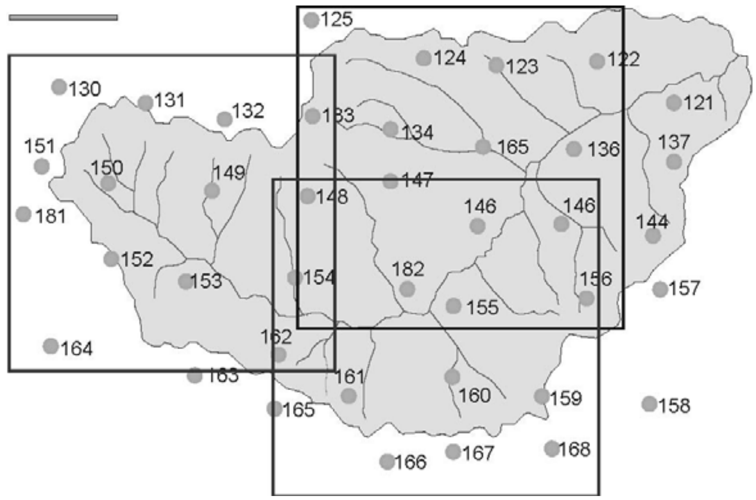

Fig. 3. A layout of the Little Washita Micronet with the three rectangular areas of about 19 km by 18 km that are used for testing the CDT method. Rain gauges within each area provide approximations of the true areal rainfall.

1. Construct the sample of concurrent point and areal rainfall for a specified spatiotemporal scale. The R_a values are approximated by averaging the rain gauge observations within the area of interest, whereas the R_p values come from all the individual gauges.

2. Estimate the sample variances of the point and areal rainfall values. Estimate the Hermite expansion coefficients for the point rainfall and the value of the scaling coefficient.

3. Perform the distribution transformation procedure described in section 2 to retrieve the areal rainfall distribution.

4. Compare the retrieved areal rainfall distribution against the observed one.

The estimates of the scaling coefficient, a, assumed values of about 0.6, for each of the sub-samples. The conditional quantile-quantile plots resulting from these tests are shown in Figure 4.

The solid dots in the plots show the comparisons of the quantiles corresponding to the same probability of exceedence for the transformation-based rainfall distributions as a function of the gauge-averaged (approximating the true areal) rainfall distributions. The open circles in the plots show the comparisons of the corresponding quantiles for the single-gauge rainfall distributions as a function of the gauge-averaged rainfall distributions. The transformation-based distributions are in a good agreement with the observed conditional distributions of areal rainfall and the degree of improvement of the radar rain gauge comparison can be seen from the comparison with the single rain gauge rainfall distributions. The tests confirm that the CDT method is able to retrieve the conditional distributions of the areal rainfall with quite good degree of accuracy.

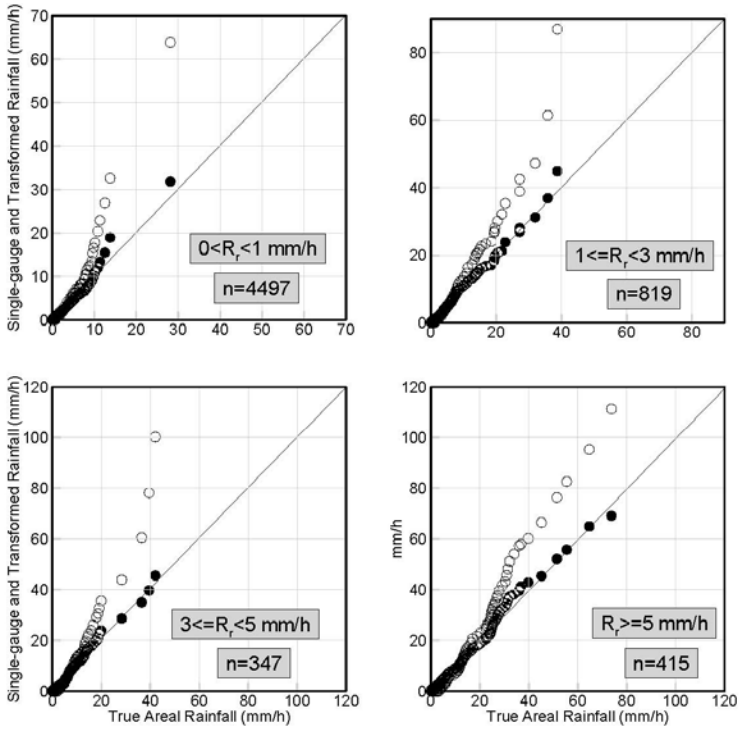

Fig. 4. A quantile-quantile plot of the cumulative rainfall distributions conditioned on the radar rainfall. Filled circles correspond to the transformation-based versus the true areal rainfall distributions. Open circles correspond to the point (single-gauge) versus the true areal rainfall distributions. The sample was stratified into four ranges of RR estimates. In the plots, n refers to the sample size in each range.

B.4.2.3. *Discussion*

The conditional scaling factors in the CDT method and their dependence on the spatial averaging scale are determined only by the distributions and the spatial correlation functions of the radar-conditioned rain gauge rainfall. These quantities are, in principle, measurable and no other fundamental assumptions are necessary to use the CDT in practical applications. However, successful application of the CDT method requires sufficiently accurate information on the rainfall spatial correlation structure conditioned on the radar estimates over the spatial scales below the resolution of the RR product. This information is available in many situations where dense rain gauge clusters exist within the sparse networks (e.g., in Oklahoma, or Iowa). Of course, the estimates of spatial correlation are bound to be uncertain. The effects of these uncertainties on the CDT scheme are complex and their quantification remains to be investigated. However, one thing that we can be sure of is that, whatever are the uncertainties in the conditional correlations, the CDT always reduces the RR uncertainty bounds in comparison with what we could obtain if we treated the single-gauge data as the corresponding truth. By its mathematical nature, the CDT just cannot increase the estimated RR uncertainty bounds. Of course, these RR uncertainty estimates are also not perfectly accurate, but this gauge-error filtering method always corrects them in the right direction. The effectiveness of this correction is clearly demonstrated in Figure 4 by comparison of its results and the single-gauge performance. As one can see, the departures from the one-to-one line for the CDT transformed and the true areal rainfall distributions are about ten time smaller than the discrepancies between the single-gauge and true areal rainfall distributions. Error reduction by an order of magnitude is a very good performance for a relatively simple statistical method.

C. SOFTWARE SETUP

Here we describe our preparations for dealing with massive radar data sets required by the PQPE algorithm development. As we argued in Krajewski and Ciach (2003) and Krajewski et al. (2003), for the PQPE algorithm to be meaningful it must be strongly based in empirical data. Fundamentally, two data types are required for this: (1) radar-rainfall estimates and (2) rain gauge data. Below we discuss our progress thus far in data collection and preparation.

C.1.1. Preparation of the Large Data Sample

Substantial efforts have been already made to prepare a large data-sample of the PPS products and the corresponding rain gauge data. This includes initiating the work on the ORPG software system, organizing a 6-year long archive of the Level II radar data from the Oklahoma City NEXRAD station (KTLX), and preprocessing of the corresponding rain gauge data from the ARS Micronet that has been acquired through the Office of Hydrology.

C.1.1.1. *The PQPE Computer and ORPG Software*

A graduate student, Mr. Gabriele Villarini, and Dr. Krajewski attended a basic training in managing the ORPG software. This training was organized by the Office of Hydrology (OH) at the NWS headquarters at the beginning of May, 2004. On June 16, a computer containing the Built 4 of the ORPG was shipped by the OH staff to the University of Iowa to be used for the PQPE project. Since then, several problems have been solved, with the help of the NWS specialists, concerning the installation of the system in our network environment and connecting the ORPG data processing to our Level II data archive. Specifically, handling some of the errors in the radar data has been tested, reading the chronological order of the data has been corrected, and the organization of the ORPG product archive has been established. We expect that other technical problems will be revealed during the automatic processing of the large radar data sample.

C.1.1.2. *The 6-year Sample of the KTLX Data*

During the previous phases of this project, large samples of the archived Level II radar data from several NEXRAD stations have been acquired from NCDC. Currently, our efforts focuse on the data from the Oklahoma City station (KTLX). This sample consists of about 350,000 data files and contains radar observations collected during the years of 1998-2003. Recently, all these files have been converted from the standard UNIX compression format that is still used by NCDC to the much more efficient “bzip” format that is currently used as a standard by the OH. We decided to adapt to this standard for its speed and efficient use of disk space.

A quality check of all the files has been performed and revealed occasional errors in the file structure. About 7,000 of the files are affected by these errors. The impact of these errors on the automatic ORPG data processing has been tested and discussed with the OH specialists. We expect that more data-error problems can be revealed during the automatic processing of this large radar data sample.

C.1.1.3. *Corresponding ARS Micronet Data*

The rain gauge data corresponding to the KTLX data sample have been acquired through the Office of Hydrology. So far, we obtained the 5-minute rain gauge accumulation data from the ARS Micronet consisting of 42 stations covering the Little Washita watershed that is located about 90 km south-west from the KTLX station. The only limitation of the analysis that can be based on these ground reference is that it covers only a very limited range of the distances from the radar (from about 75 km to about 105 km). Therefore, it is essential that the Office of Hydrology helps us to acquire also the Oklahoma Mesonet (Brock et al. 1995) data that cover the full range of distances.

The original ARS Micronet archive is organized in a very inefficient way and consists of about 700,000 small files. In addition, the timing convention in this archive is incompatible with the radar data. To make it usable, we preprocessed the archive converted it to 60 monthly files of the 5-minute rain gauge rainfall. During this preprocessing, several errors in the data were detected. These errors were corrected, whenever possible, or flagged as missing data records.

D. UNCERTAINTY IN FLASH-FLOOD GUIDANCE SYSTEM

Demonstration of the benefits from improved prediction of flash-floods is the main hydrologic application we focus on. Here the main challenge is the timely issuing of the flash-flood warnings (e.g. Georgakakos et al. 1997). The current technology used by the NWS is based on the concept of threshold rainfall (Carpenter et al. 1999; Carpenter et al. 2001; Reed et al. 2002), i.e. rainfall amount that will cause flooding (exceedence of bank-full discharge). This amount depends on static variables (i.e. changing slowly with time) such as land use, topography, channel network, etc. and dynamic variables (fast changing in time) that include soil moisture, snow melt, ground temperature. The threshold value that corresponds to the average (i.e. typical) conditions and determined based on the static variable, is adjusted in real time based on the monitoring of the dynamic variables. Collectively these are know as flash flood guidance (FFG) procedures and are used on a basin by basin basis. They are used operationally by the NWS Weather Offices around the country using hydrologic models operated by the River Forecast Centers. When rainfall amount integrated over the basins exceeds the threshold value a warning to the public is issued. Although the analysis is done based on basin delineation (i.e. in a hydrologically meaningful way) the warnings are issued on a county basis with which the public is more familiar. Typically the basins that are subject to FFG analysis are smaller that a typical county. Our demonstration, described in Appendix A, is for the Oklahoma region (Figure 5).

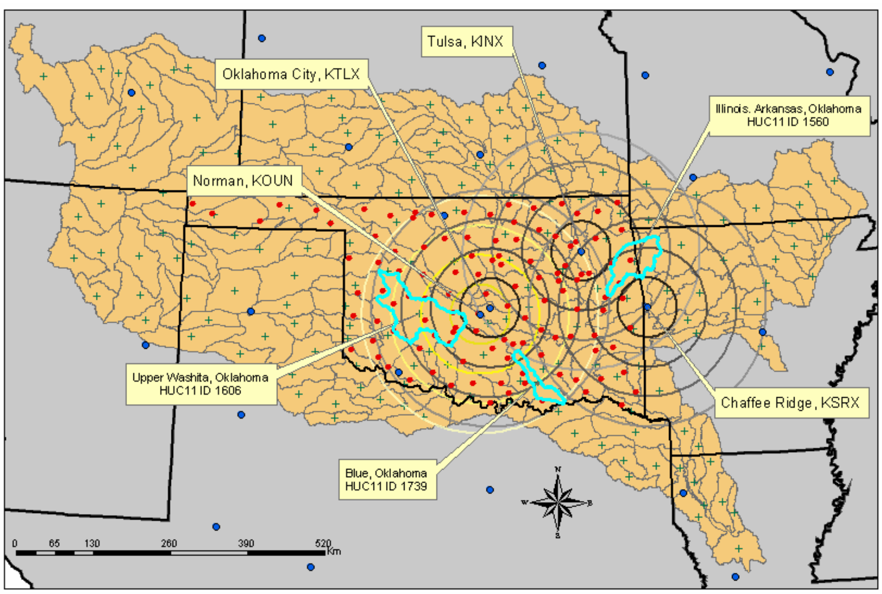

Figure 5. Arkansas River Basin with its radar coverage (blue dots and 50 km spaced rings), Oklahoma Mesonet (red dots), polarimetric WSR-88D (yellow rings), and basins selected as potential study sites.

Our demonstration, described in Appendix A, is for the Oklahoma region (Figure 5).

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