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Diagnosing Quasi-Geostrophic Forcing Using PC-GRIDDS: A Case Study

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1. INTRODUCTION

Until recently, forecasters have had limited means to diagnose quasi-geostrophic (Q-G) forcing. The limited graphical model output (from AFOS or standard pressure level graphics) available makes diagnosing Q-G forcing difficult in certain situations (Sanders and Hoskins 1990; Dunn 1991). Current operational practices for estimating Q-G forcing are based on pre-NWP and early-NWP era techniques and are outdated (Dunn 1991). Barnes (1985) stated that operational diagnosis of Q-G forcing has been plagued by approximations and assumptions.

Typically, forecasters rely on thickness and vorticity graphics that are available over AFOS, or standard pressure level graphics, to **qualitatively** estimate Q-G forcing. Durran and Snellman (1987) showed that qualitative approximations of Q-G forcing fail, at times, to give an accurate sense of Q-G vertical motion. Wiesmueller and Brady (1993) presented a case study in which qualitative estimates of Q-G forcing, based on graphical model output, failed to capture the significant Q-G forcing present in a Mid-Atlantic States heavy rain event. Potential errors in determining the sense of Q-G vertical velocity justifies the need for **quantitative** diagnoses of Q-G forcing.

The PC-GRidded Interactive Diagnostic and Display System software (PC-GRIDDS) gives forecasters the tools to **quantitatively** diagnose Q-G forcing. PC-GRIDDS is a software package that assimilates model gridded data, allowing forecasters to generate and display a wide variety of meteorological fields. With this software package, forecasters can look at derived fields, useful for diagnosing Q-G forcing, that are not available over the National Weather Service AFOS system or from standard pressure level graphics. The reader is referred to Dunn (1991) for more details regarding the use of gridded model output for diagnosing Q-G forcing and other dynamical processes.

Use of gridded data software programs, like PC-GRIDDS, is essential for accurately diagnosing Q-G forcing in an operational forecast office (Dunn 1991). Wiesmueller and Brady (1993) showed that derived fields from gridded data, useful for diagnosing Q-G forcing, provided better insight in interpreting the vertical motion patterns with a Mid-Atlantic States rain event. Both Dunn (1991) and Wiesmueller and Brady (1993) make strong cases for the potential usefulness of gridded data software programs, like PC-GRIDDS, in operational forecast offices.

After an overview of the Q-G theory for determining vertical motion, this paper will present a case study that **demonstrates** the usefulness of PC-GRIDDS for diagnosing Q-G forcing. The case study is of a heavy snow event that occurred in northern lower Michigan, for which qualitative estimates of Q-G forcing based on the vorticity and thickness fields from AFOS (or standard pressure level graphics) failed to account for the model vertical motion. PC-GRIDDS-derived fields of differential vorticity advection, the Laplacian of thermal advection, and Q-vector divergence were used to diagnose the Q-G vertical velocity. These PC-GRIDDS-derived fields supported the model vertical motion, proving to be a better representation of the Q-G forcing in this case.

2. USING QUASI-GEOSTROPHIC THEORY TO DIAGNOSE VERTICAL MOTION

It is well known that the synoptic-scale vertical motion field plays a significant role in producing large-scale cloud systems and precipitation. As a consequence of this, operational meteorologists heavily rely on the evolution of synoptic-scale vertical velocity fields for forecasting sensible weather. Since the quasi-geostrophic vertical velocity is a good approximation to the total vertical velocity in synoptic-scale systems (Durran and Snellman 1987), a thorough understanding of Q-G theory is important. The importance of understanding Q-G theory has been cited in work by Hoskins et al. (1978), Barnes (1985, 1986 and 1987), Durran and Snellman (1987), and more recently Dunn (1991).

Q-G forcing is the adjustment process that takes place when thermal wind balance is destroyed by geostrophic advection (Durran and Snellman 1987). The adjustment process requires vertical motion to adiabatically cool or warm the atmosphere in order to restore thermal wind balance (Durran and Snellman 1987). This vertical motion is often referred to as Q-G vertical velocity. Q-G vertical velocity is computed from the classic omega equation, which has two forcing terms.

$$\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega = \underbrace{-\frac{f_0}{\sigma}\frac{\partial}{\partial p}[-\mathbf{v}_g \cdot \nabla_p(\zeta_g + f)]}_{A} + \underbrace{\frac{R}{\sigma p}[-\nabla_p^2(-\mathbf{v}_g \cdot \nabla_p T)]}_{B} \qquad (1)$$

(Note that mathematical symbols and meteorological variables used in this paper have their conventional meaning and are defined in the Appendix.)

Term A is the vertical derivative of the geostrophic vorticity advection and term B is the horizontal Laplacian of thermal advection. Diabatic heating, orographic lift and friction also affect vertical motion, but on the synoptic-scale they are about an order of magnitude less than terms A and B, and are typically scaled out. A more detailed description of the omega equation can be found in Carlson (1991), Holton (1992), and Bluestein (1992).

Due to the limited graphical fields that are available from AFOS or from standard pressure level graphics, forecasters rely on qualitative approximations of terms A and B of the omega equation for diagnosing Q-G forcing. Typically, term A is approximated by estimating vorticity advection at 500 mb from the 500 mb height and vorticity fields. Similarly, term B of the omega equation is approximated by estimating thermal advection from the 850 mb height and temperature fields, or the MSL pressure and 1000 to 500 mb thickness fields. These estimates date back to the pre-NWP and early-NWP eras, and are still commonly used in operational forecasting due to the limited graphical fields that were previously available (Dunn 1991).

Durran and Snellman (1987) have shown that there are problems with estimating Q-G forcing in this manner. First, the estimate of term A accounts for the vorticity advection at one level (500 mb), totally ignoring the vertical derivative. Second, the estimate of term B accounts for the thermal advection field only, ignoring the Laplacian operator entirely.

The importance of the vertical derivative in Term A is illustrated in the following examples. Negative vorticity advection (NVA) at 500 mb forces upward vertical motion if stronger NVA is present below 500 mb. This is because NVA decreasing with height actually results in positive differential vorticity advection. In the same way, positive vorticity advection (PVA) decreasing with height results in negative differential vorticity advection, leading to subsidence. Durran and Snellman (1987) demonstrate this by presenting a case in which 500 mb negative vorticity advection (NVA) led to upward motion since the vertical derivative of vorticity advection was positive. In weaker flow regimes, typical of the warm season, the strongest Q-G forcing often occurs above 500 mb (Wiesmueller and Brady 1993), rendering the 500 mb vorticity advection field useless for diagnosing Q-G forcing.

Durran and Snellman (1987) also illustrated the problems that can occur from ignoring the Laplacian operator in term B of the omega equation. They showed areas within a warm advection (WAA) field that led to subsidence due to the effects of curvature in the WAA field. Bluestein (1992) demonstrates how the Laplacian of temperature advection forces upward motion in regions where there is a local maximum in the temperature advection field, **regardless of the sign of the advection**. Likewise, a local minimum in the temperature advection field leads to subsidence **regardless of the sign of the advection**. For example, a weakness in a cold air advection (CAA) region is mathematically a local maximum which supports upward motion. Similarly, a weakness in a WAA region is a local minimum leading to subsidence.

Another significant problem with qualitative approximations of Q-G vertical velocity is that cancellation often occurs between term A and term B of the omega equation (Trenberth 1978; Hoskins et al. 1978; Durran and Snellman 1987; Barnes 1985, 1986 and 1987). For example, cancellation between the two terms of the omega equation often occurs downstream of an upper-level ridge axis where WAA and

NVA are found, and downstream from the upper-level trough axis where CAA and PVA are found. Cancellation between terms A and B of the omega equation makes qualitative assessments of Q-G forcing difficult since it is hard to determine the relative strength of each term.

One solution to the problem of cancellation is presented in Trenberth (1978). Trenberth rewrote the right hand side (RHS) of the omega equation and derived the following equation which combines the forcing from terms A and B of the omega equation.

$$\left(\nabla_p^2+\frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega=\frac{f_0}{\sigma}\Big[\underbrace{2\left(\frac{\partial \mathbf{v}_g}{\partial p}\cdot\nabla_p\zeta_g\right)}_{A}+\underbrace{\frac{\partial \mathbf{v}_g}{\partial p}\cdot\nabla_p f}_{B}-\underbrace{2D^2\frac{\partial\theta_D}{\partial p}}_{C}\Big]. \qquad (2)$$

Term A of Eq. (2) is the geostrophic vorticity advection by the thermal wind in a layer; term B is the beta term and term C accounts for thermal deformation. Terms B and C are usually scaled out, leaving only term A. Thus, the Trenberth approximation states that the geostrophic vorticity advection by the thermal wind in a layer is approximately equal to the forcing from the RHS of the omega equation. This approximation accounts for the canceling effects of the differential vorticity advection (term A) and Laplacian of thermal advection (term B) terms in the classic omega equation (Eq. (1)).

The Trenberth approximation is operationally useful because it can be applied **qualitatively** to standard pressure level graphics and AFOS graphics. Forecasters apply the Trenberth approximation by advecting the 500 mb vorticity by the 1000 to 500 mb thickness field. Aside from being a qualitative method, one significant shortcoming to this approach is that the 500 mb vorticity fields (the only one available over AFOS or from standard pressure level graphics) may not reflect the mean layer vorticity from 1000 to 500 mb, leading to errors in determining the sign of the Q-G vertical velocity (Barnes 1985).

Another solution to the problem of cancellation of terms A and B of the omega equation is described in Hoskins et al. (1978). Similar to Trenberth (1978), Hoskins et al. reformulated the classic omega equation (Eq. (1)) to account for the canceling effects of the RHS terms, and developed the concept of Q-vectors. It can be shown that the RHS of the omega equation can be written as

$$\left(\nabla_p^2+\frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega=-2\nabla_p\cdot\mathbf{Q}, \qquad (3)$$

where **Q** are Q-vectors. Equation (3) states that the divergence of **Q** is proportional to the left hand side of the omega equation, and is the combination of both terms A and B of the omega equation (Eq. (1)). Hoskins et al. (1978) show that Q-vector convergence leads to upward vertical motion, and Q-vector divergence leads to subsidence. Even in areas where terms A and B tend to cancel, Q-vector divergence fields yield an accurate assessment of Q-G vertical motion. In addition, Q-vector divergence fields account for the thermal deformation term that is scaled out in the Trenberth approximation.

The PC-GRIDDS software package allows one to calculate differential vorticity advection, Laplacian of the temperature advection, and Q-vector divergence. These PC-GRIDDS Q-G diagnostic calculations are **quantitative** and provide a more reliable representation of the forcing from the RHS of the omega equation, when compared to the older qualitative assessment methods. The next section of this paper demonstrates the advantages of using these PC-GRIDDS fields over qualitative estimates of Q-G forcing from graphical model output.

3. CASE STUDY: JANUARY 28, 1994 SNOWSTORM

On January 28, 1994, a snowstorm blanketed northern lower Michigan with a band of 6 to 10 inches of snow (Fig. 1). Although snowfall amounts with this storm were not that impressive, snowfall rates were. Most of the snow fell in an 8-hour period from 2000 UTC to 0400 UTC, January 28-29 1994, yielding snowfall rates of more than 1 inch per hour in some areas. NGM 12-hour forecast graphics valid 0000 UTC, 29 January 1994 will be presented for this case.

The forecast 500 mb height and vorticity fields and vorticity advection field for 0000 UTC, 29 January 1994 are shown in Figure 2. Qualitatively from the 500 mb height and vorticity field alone, neutral 500 mb vorticity advection was forecast to occur during the height of the heavy snowfall (near 0000 UTC) over lower Michigan. At the same time, cold air advection (CAA) was forecast to occur across the Great Lakes region (Fig. 3). Using the 500 mb height and vorticity chart and MSL pressure and thickness chart that would be available over AFOS or from standard pressure level graphics, a forecaster may conclude that little Q-G forcing was progged over Michigan at 0000 UTC.

The forecast 700 mb vertical velocities for 0000 UTC, 29 January 1994 reveal a different picture, with significant upward vertical velocities across lower Michigan (Fig. 4). It is apparent that the qualitative estimates of Q-G forcing were unable to account for the significant upward vertical motions produced by the model for this case. The remainder of this section will present quantitative assessments of Q-G forcing from PC-GRIDDS-derived fields.

The forecast 850 mb height and vorticity fields and vorticity advection field (Fig. 5) at 0000 UTC shows 850 mb NVA over northern lower Michigan. This NVA was forecast to occur underneath weak 500 mb vorticity advection as indicated in Figure 2, which results in positive differential vorticity advection over northern lower Michigan. Figure 6 is a PC-GRIDDS display of differential vorticity advection for the layer from 850 to 300 mb valid 0000 UTC, January 29, 1994. Clearly, **positive differential vorticity advection** was forecast over northern lower Michigan. Thus, it can be concluded that term A of the omega equation was forecast to contribute to upward vertical motion despite neutral vorticity advection at 500 mb.

Figure 7 shows the forecast 700 mb temperature advection field valid 0000 UTC, 29 January 1994. A weakness in the CAA field is apparent over lower Michigan, which is a local **maximum** in the temperature advection field. This local maximum leads to negative values in the forecast 700 mb Laplacian of this field over lower Michigan (Fig. 8), supporting upward vertical motion. Thus despite CAA, it can be concluded that term B of the omega equation also contributed to upward motion over northern lower Michigan.

Although it appeared from initial qualitative assessments that both terms of the omega equation were either weak or contributing to subsidence, in reality, both terms were contributing to upward vertical velocities. The 700 mb Q-vector divergence field forecast valid for 0000 UTC, January 29, 1994 was calculated to illustrate the combined forcing from both terms in the omega equation (Fig. 9). Not surprisingly, a swath of Q-vector convergence was indicated at 700 mb over northern lower Michigan. This is consistent with not only the forcing from terms A and B of the omega equation but also the 700 mb vertical motion predicted by the model.

Wiesmueller and Brady (1993) mention that layer-averaged Q-vector divergence fields often provide a better representation of Q-G forcing than the Q-vector divergence fields calculated at one level. The 850-500 mb layer averaged Q-vector divergence field was calculated for this case (not shown) using PC-GRIDDS, and provided similar results to the 700 mb Q-vector divergence field, showing Q-vector convergence across northern Lower Michigan.

4. CONCLUSION

It is apparent that qualitative assessments of Q-G forcing from the 500 mb height and vorticity fields and MSL pressure and thickness fields (or 850 mb height and temperature fields) can fail to reveal the Q-G forcing present. The main problems that arise from these qualitative estimates are that: 1) estimating term A of the omega equation from the 500 mb height and vorticity fields does not account for the vertical derivative of vorticity advection, 2) estimating term B from the MSL pressure and 1000 to 500 mb thickness fields (or 850 mb height and temperature fields) ignores the Laplacian operator in term B, and 3) cancellation between the two terms of the omega equation makes it difficult to determine the sign of the Q-G vertical velocity.

This case study illustrates the inaccuracies that can arise from qualitative estimates and gives credence to the use of PC-GRIDDS-derived fields of differential vorticity advection, Laplacian of temperature advection, and Q-vector divergence for diagnosing Q-G forcing. These PC-GRIDDS-derived fields provide a more reliable representation of the Q-G forcing since they take into account both the vertical derivative in term A and the Laplacian operator in term B of the omega equation.

PC-GRIDDS is useful not only for diagnosing Q-G forcing, but also for diagnosing conditional symmetric instability, jet streak dynamics and associated ageostrophic circulations (Dunn 1991) and severe weather potential. Wiesmueller and Brady (1993) show the usefulness of PC-GRIDDS for diagnosing lift on isentropic surfaces and displaying cross-sections depicting ageostrophic secondary circulations.

As the National Weather Service moves to the AWIPS-era, forecasters will have the ability to do a variety of model grid manipulations to assess various operational forecast problems. Using PC-GRIDDS **now** will facilitate the transition from AFOS to AWIPS by increasing forecaster skill in the arena of model grid manipulations.

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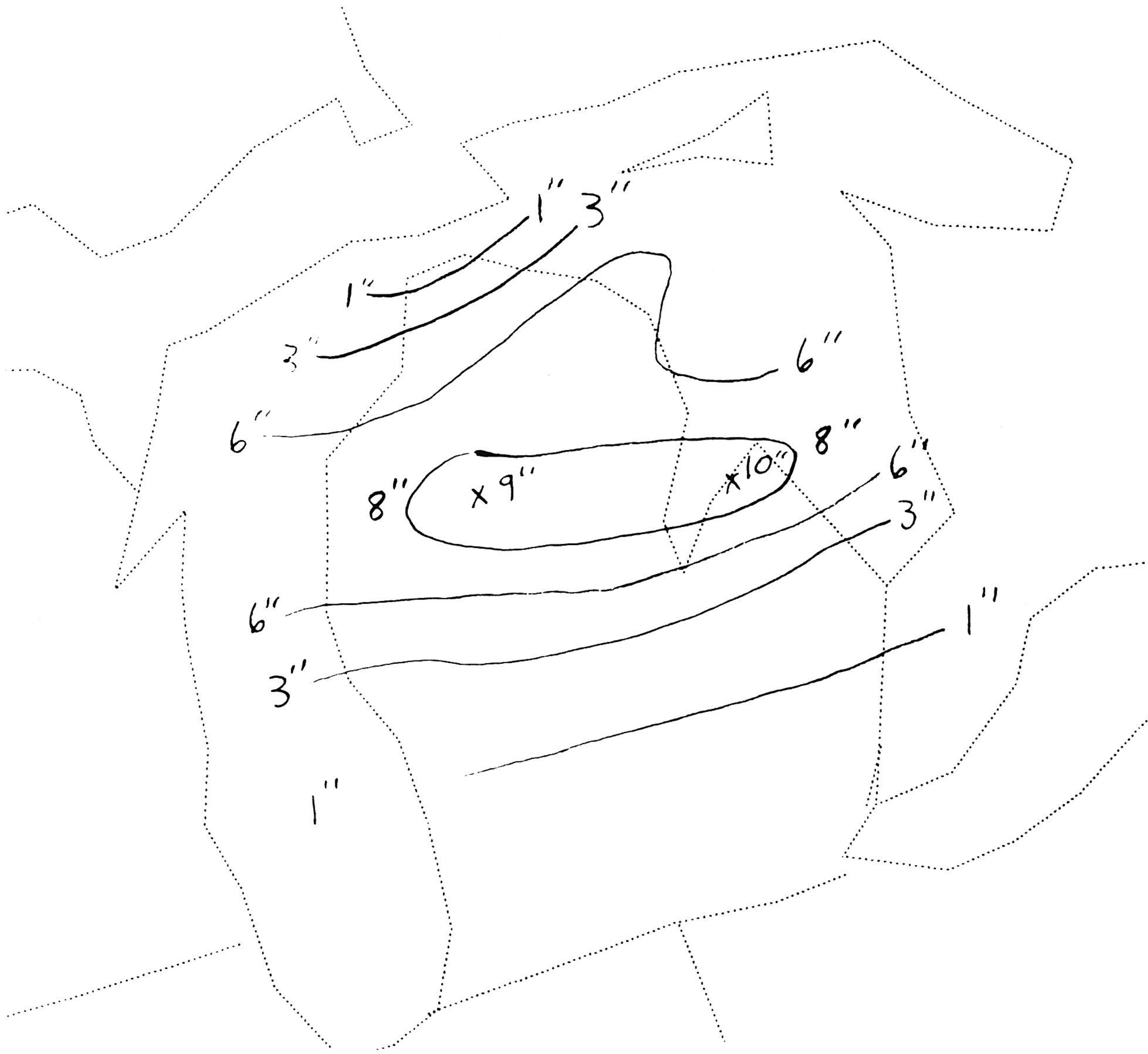
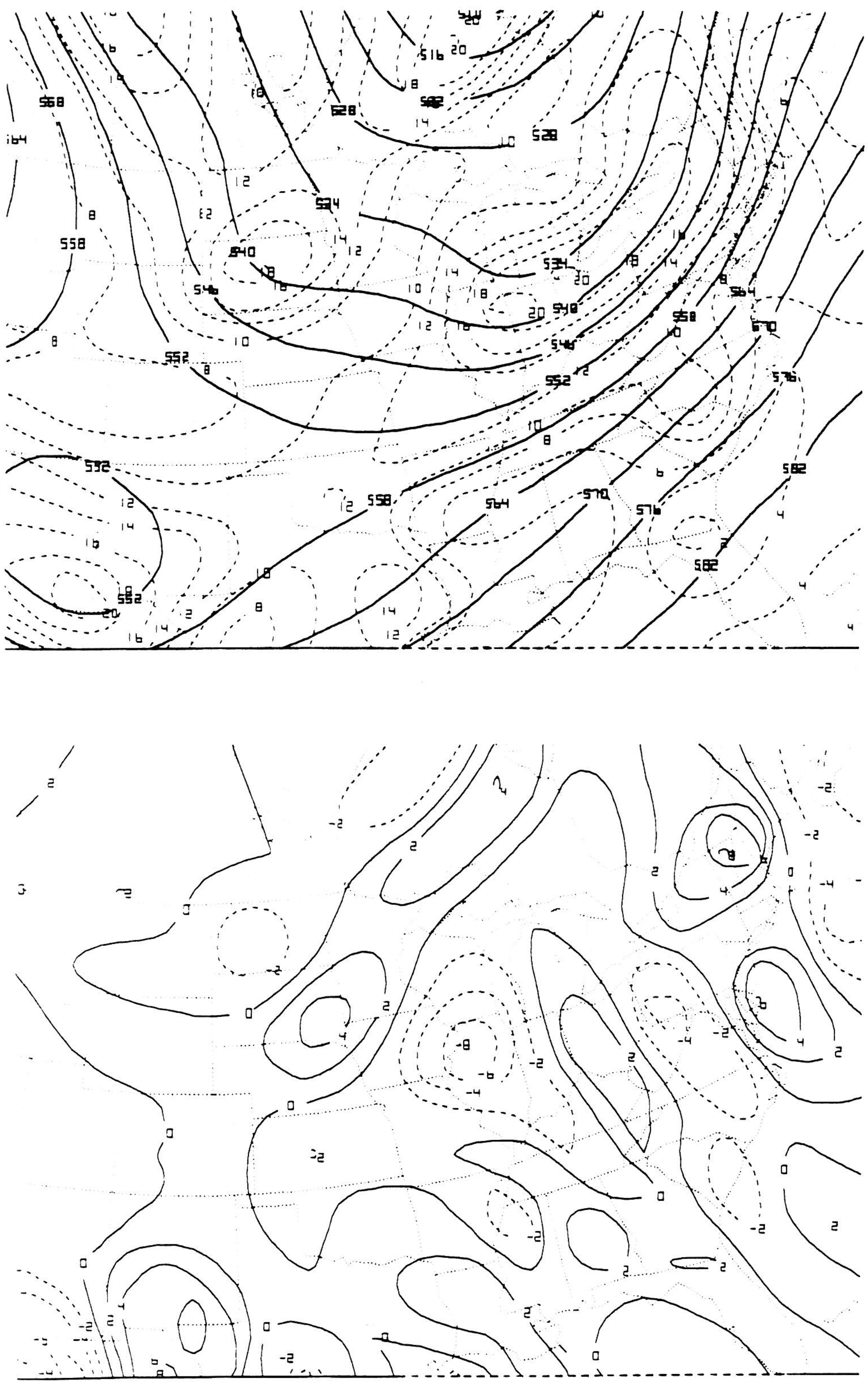


Figure 1. Snowfall distribution across northern lower Michigan January 28, 1994. Contours in inches.



(a)

(b)

Figure 2. Twelve-hour NGM forecast valid 0000 UTC, 29 January 1994 showing a) 500 mb height (solid, dm) and vorticity (dash, 10^{-5} sec^{-1}), and b) 500 mb vorticity advection (10^{-9} sec^{-2}).

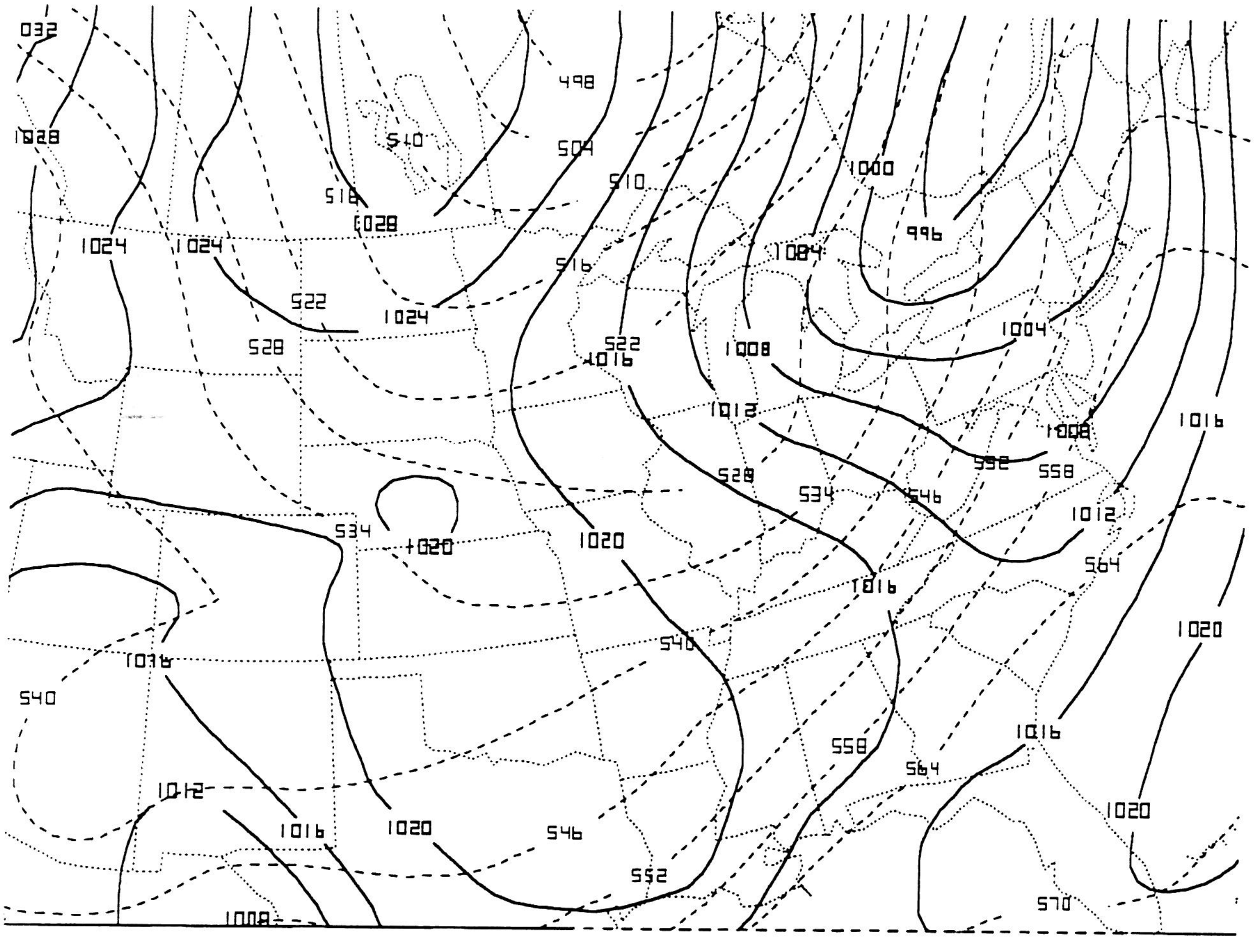


Figure 3. Twelve-hour NGM forecast of MSL pressure (solid, mb) and 1000 to 500 mb thickness (dash, dm) valid 0000 UTC, 29 January 1994.

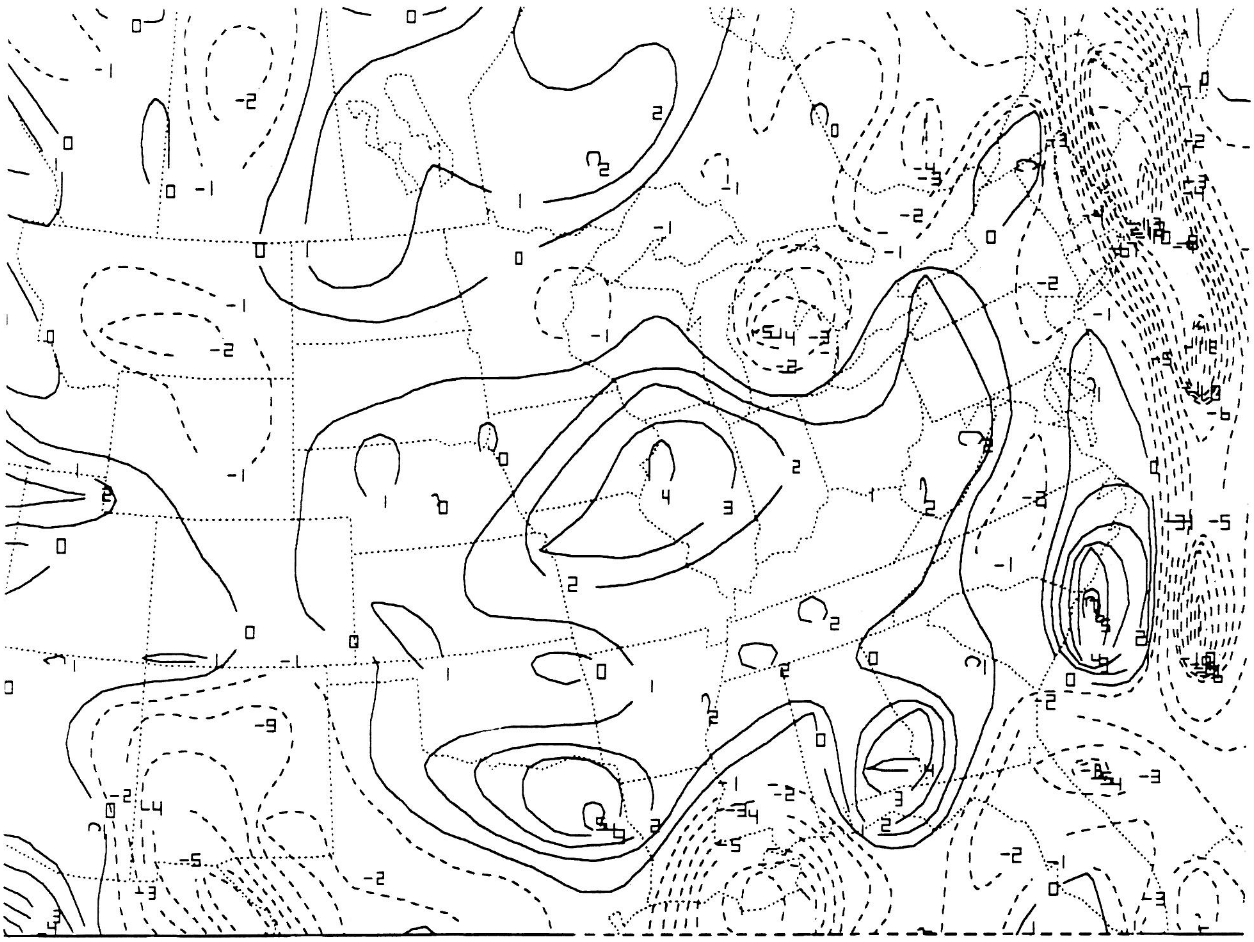
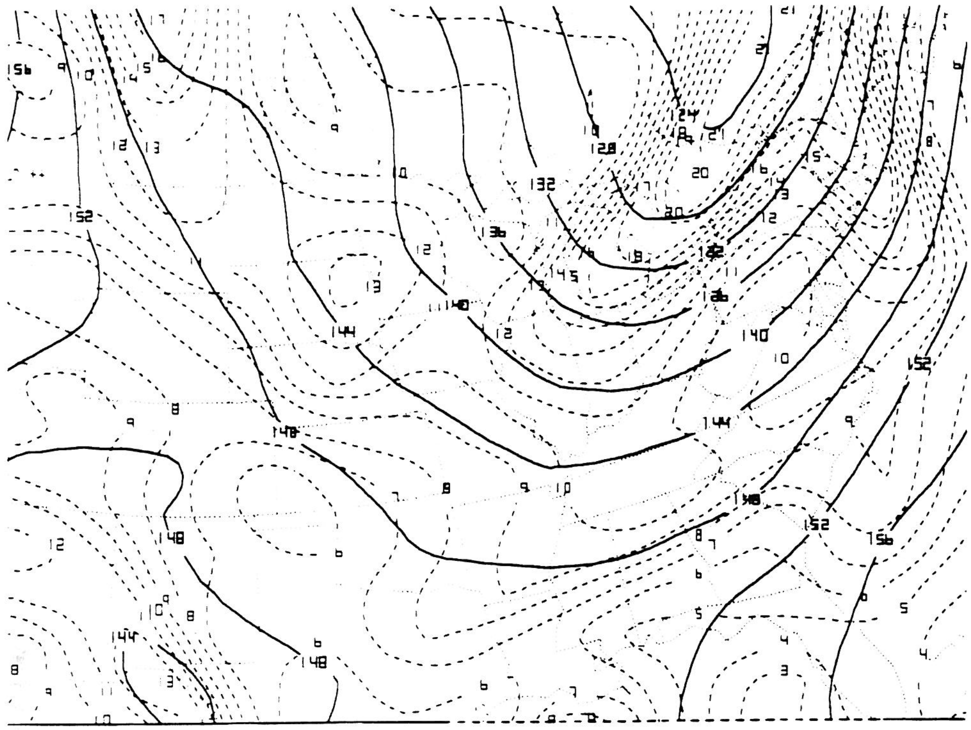
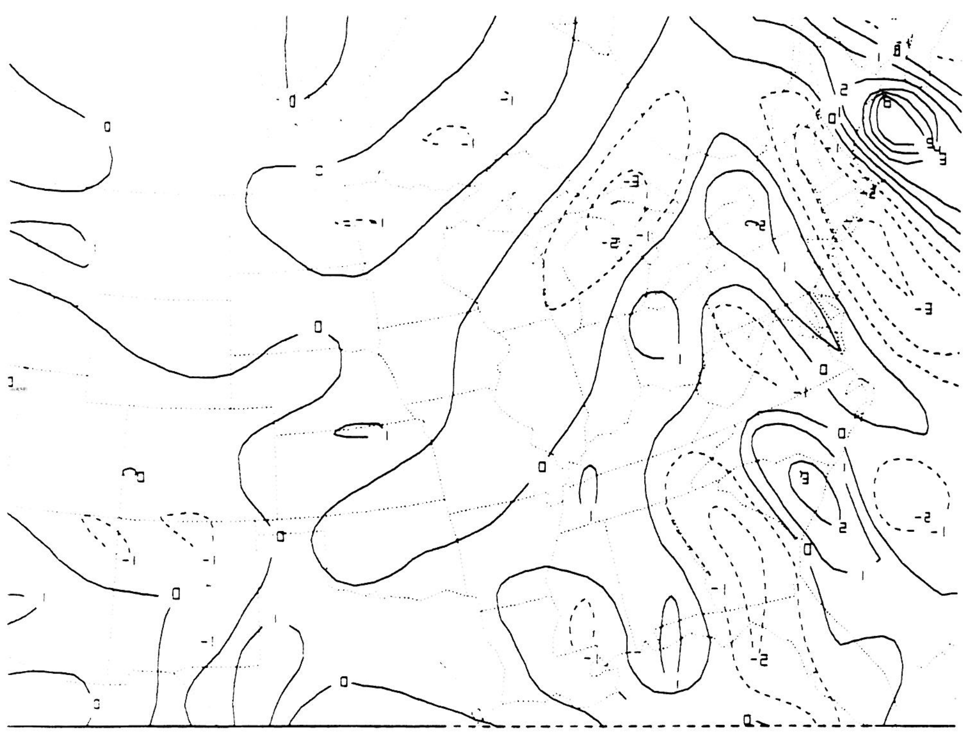


Figure 4. Twelve-hour NGM forecast of 700 mb vertical velocity (microbar/sec) valid 0000 UTC, 29 January 1994. Dashed (negative) values denote upward vertical motion.



(a)



(b)

Figure 5. As in figure 2 except for 850 mb.

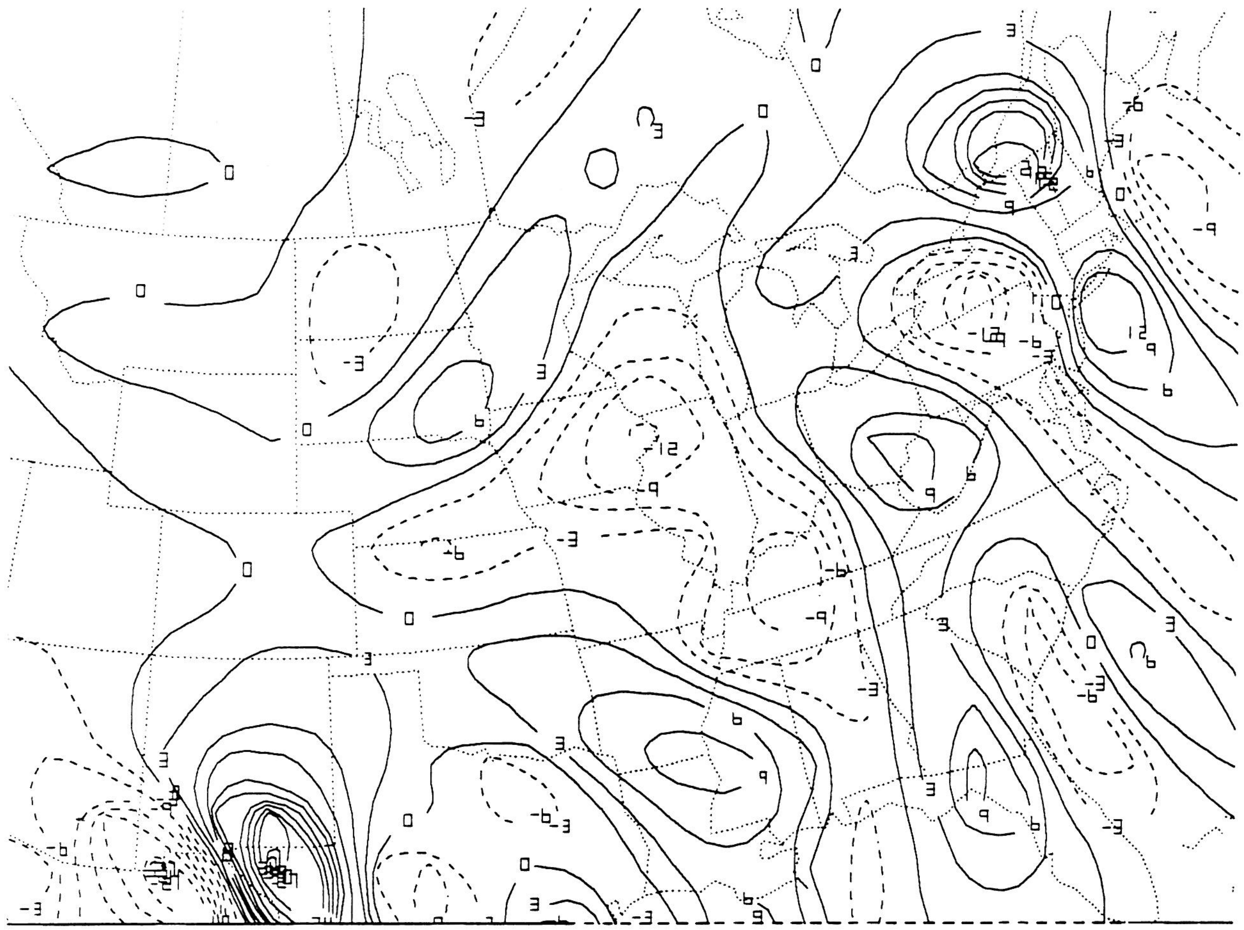


Figure 6. Twelve-hour NGM forecast of differential vorticity advection (10^{-9} sec^{-2}) in the layer 850 mb to 300 mb valid 0000 UTC, 29 January 1994.

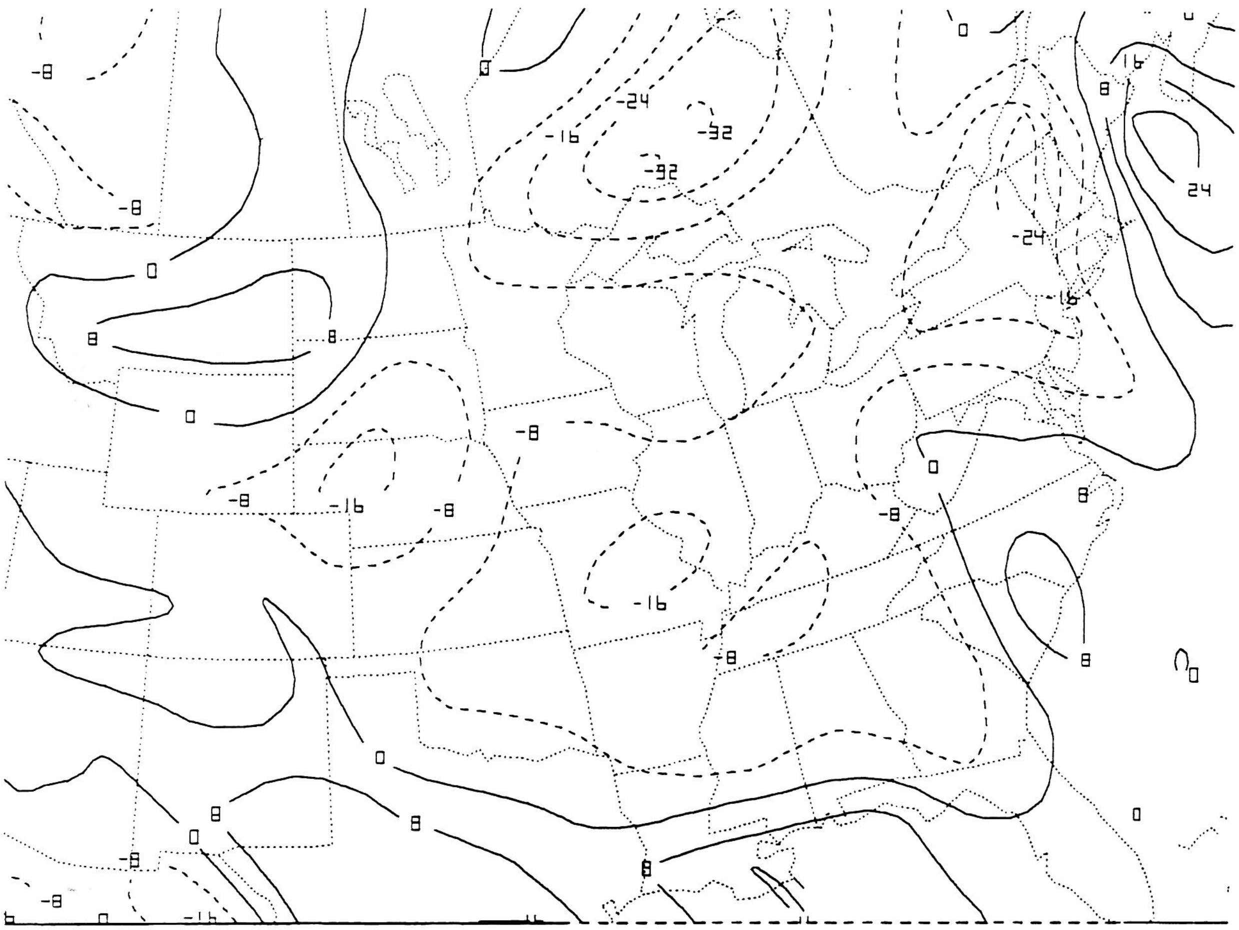


Figure 7. Twelve-hour NGM forecast of 700 mb temperature advection (10^{-4} °C/sec) valid 0000 UTC, 29 January 1994.

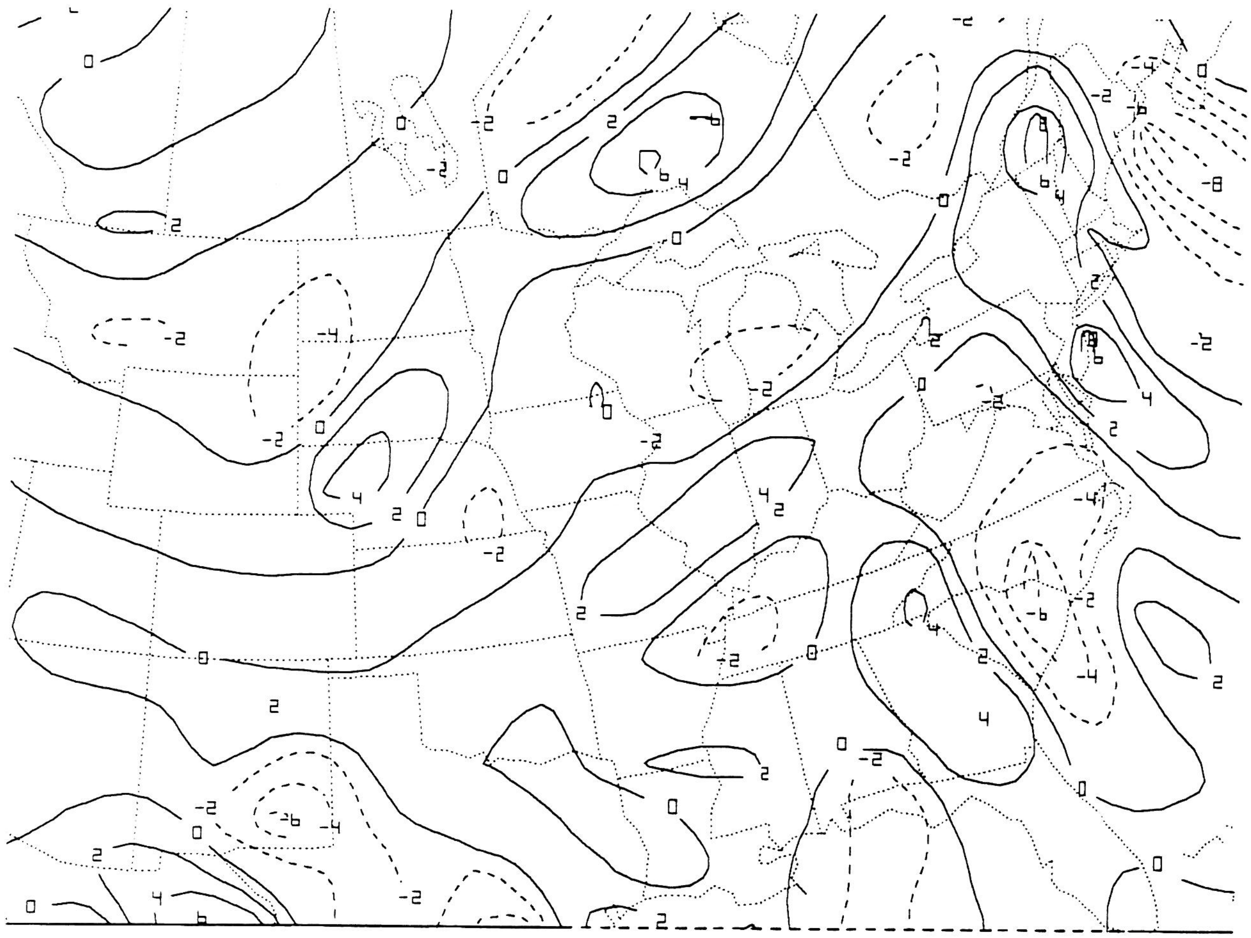


Figure 8. Twelve-hour NGM forecast of 700 mb Laplacian of the temperature advection (10^{-14} $^{\circ}C/m^2$-sec) valid 0000 UTC, 29 January 1994. Dashed (negative) values support upward vertical motion.

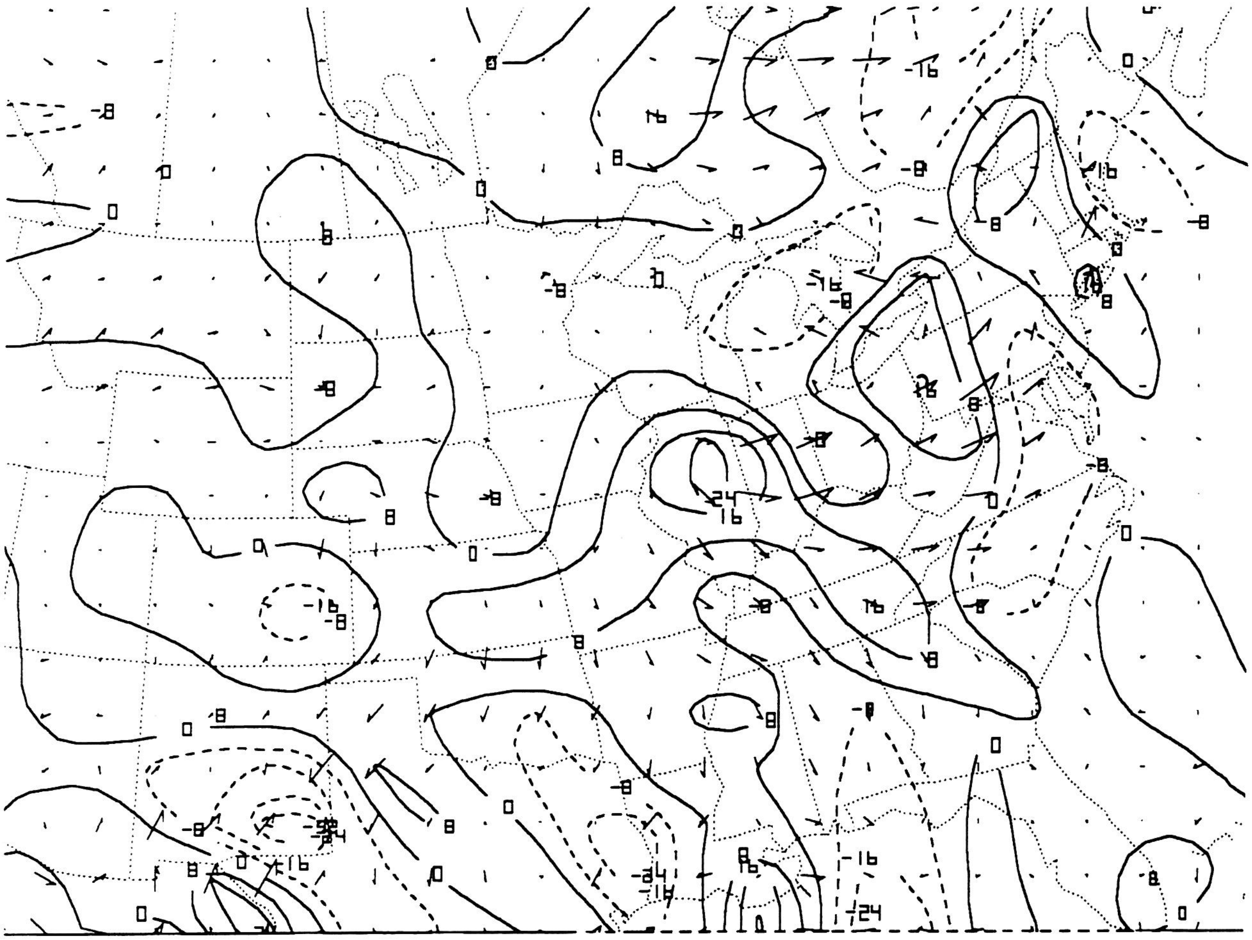


Figure 9. Twelve-hour NGM forecast of 700 mb Q-vectors and Q-vector divergence (10^{-15} m-kg^{-1} sec^{-1}) valid 0000 UTC, 29 January 1994. Dashed (negative) values denote convergence.

APPENDIX

∂	Partial Derivative
∇_p	Horizontal gradient operator on a constant pressure surface
∇_p^2	Two-dimensional Laplacian operator on constant pressure surface
f	Coriolis parameter
ζ	Vorticity (vertical component)
ω	Vertical velocity in isobaric coordinates
σ	Static stability parameter
R	Gas Constant for dry air
θ_D	The angle formed between the axis of dilation or contraction and the x-axis.

$$\mathbf{D}_g = \begin{pmatrix} D_1 \\ D_2 \end{pmatrix} \qquad D_1 = \frac{\partial u_g}{\partial x} - \frac{\partial v_g}{\partial y}$$

$$D_2 = \frac{\partial v_g}{\partial x} + \frac{\partial u_g}{\partial y}$$

Q	Q-vector	where $\mathbf{Q} = -\frac{f_0}{\sigma} \zeta_g \frac{\partial \mathbf{v}_g}{\partial p}$
p	Pressure	
V_g	Geostrophic wind vector	